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B.A./B.Sc. THIRD SEMESTER EXAMINATION, DECEMBER 2015
SECOND YEAR [BATCH 2014-17]
STATISTICS (General)
Paper : III

Date : 21/12/2015
Time : 11 am – 1 pm
Full Marks : 50

[Use a separate Answer Book for each group]

Group – A

Answer **any two** questions : (2×5)

1. Let $X_1, X_2, ..., X_n$ be a random sample from N(0,1). Define $\bar{X}_k = \frac{1}{k}\sum_{i=1}^{k} X_i$ and $\bar{X}_{n-k} = \frac{1}{n-k}\sum_{i=k+1}^{n} X_i$.
   Find the distributions of $Y_k = ½(\bar{X}_k + \bar{X}_{n-k})$ and $Z_k = k\bar{X}_k^2 + (n-k)\bar{X}_{n-k}^2$. (2+3)
2. Let *X* & *Y* denote the number of successes and failures respectively in *n* independent bernoullian trials with *p* as the probability of success in each trial. Show that
   $$\frac{(X-np)^2}{np} + \frac{[Y-n(1-p)]^2}{n(1-p)}$$
   can be approximated by a χ^2-distribution with one degree of freedom when *n* is large.
3. If T_1 & T_2 be two unbiased estimators of parameter θ with variances σ_1^2, σ_2^2 and correlation ρ, come up with the linear combination of T_1 & T_2, which will be the best linear unbiased estimator of θ.
4. Given $P[F_{10,12} > 2.753] = 0.05 = P[F_{1,12} > 4.747]$.
   Find $P[F_{12,10} > \frac{1}{2.753}]$ and $P[-\sqrt{4.747} < t_{12} < \sqrt{4.747}]$.
   [The letters F and t denote respectively F-distribution and t-distribution. The integers affixed to F and t are the relevant degrees of freedom] (2+3)

Answer **any two** questions : (2x10)

5. a) In context of testing of hypothesis, define the following terms:
   (i) Level of significance
   (ii) Power of test.
   b) Let $X \sim N(\mu, \sigma^2 = 4)$. To test $H_0 : \mu = -1$ against $H_1 : \mu = 1$, based on a sample of size 10 from this population, we use the critical region:
   $$X_1 + 2X_2 + \cdots + 10X_{10} \geq 0 \text{ i.e. } \sum_{i=1}^{10} iX_i \geq 0$$
   What is its size? What is the power of the test? (3+7)
6. It is claimed that students entering a college have an average I.Q. higher than 100. A random sample of 16 is taken and the sample mean and standard deviation are found to be 106 and 10 respectively. Is the claim supportable? (It is assumed that the I.Q's are normally distributed.)
7. Let $X_1, X_2, ..., X_n$ be a random sample of size '*n*' from N(μ,1). Find the MLE of μ & show that it is unbiased and consistent. (5+5)

8. In statistical inference, explain the application of Pearsonian statistics for testing –
   - Goodness of fit
   - Independence of two attributes. (5+5)

Group – B

Answer **any two** questions : (2×5)

9. What do you mean by trend? Describe the method of least square to determine trend.
10. Define a Modified Exponential curve. Explain how you can fit a modified exponential curve.
11. What do you mean by chain index? Explain the advantages of chain index over fixed base index.
12. Write down the use of COLIN. Mention the steps of construction of COLIN.

Answer **any one** question : (1×10)

13. Briefly explain different methods of construction of index Number. Explain the errors in construction of index Number. Why is Fisher's index Number called the ideal index Number? (4+4+2)
14. Find the seasonal indices by method of ratio-to-moving averages from the following data:

Production of commodity (in 1000 tons)

Year / Quarter	I	II	III	IV
1973	37	38	37	40
1974	41	34	25	31
1975	35	37	35	41

TABLE IV *t*-Distribution*

Values of $t_{\alpha,\nu}$

ν \ α	0·05	0·025	0·01	0·005
1	6·314	12·706	31·821	63·657
2	2·920	4·303	6·965	9·925
3	2·353	3·182	4·541	5·841
4	2·132	2·776	3·747	4·604
5	2·015	2·571	3·365	4·032
6	1·943	2·447	3·143	3·707
7	1·895	2·365	2·998	3·499
8	1·860	2·306	2·896	3·355
9	1·833	2·262	2·821	3·250
10	1·812	2·228	2·764	3·169
11	1·796	2·201	2·718	3·106
12	1·782	2·179	2·681	3·055
13	1·771	2·160	2·650	3·012
14	1·761	2·145	2·624	2·977
15	1·753	2·131	2·602	2·947
16	1·746	2·120	2·583	2·921
17	1·740	2·110	2·567	2·898
18	1·734	2·101	2·552	2·878
19	1·729	2·093	2·539	2·861
20	1·725	2·086	2·528	2·845
21	1·721	2·080	2·518	2·831
22	1·717	2·074	2·508	2·819
23	1·714	2·069	2·500	2·807
24	1·711	2·064	2·492	2·797
25	1·708	2·060	2·485	2·787
26	1·706	2·056	2·479	2·779
27	1·703	2·052	2·473	2·771
28	1·701	2·048	2·467	2·763
29	1·699	2·045	2·462	2·756
30	1·697	2·042	2·457	2·750
40	1·684	2·021	2·423	2·704
60	1·671	2·000	2·390	2·660
120	1·658	1·980	2·358	2·617
∞	1·645	1·960	2·326	2·576

*Abridged from Table 12 of *Biometrika Tables for Statisticians*, vol. I, with the kind permission of the Biometrika Trustees.

TABLE I ORDINATES AND AREAS OF THE DISTRIBUTION OF STANDARD NORMAL VARIABLE*

τ	φ(τ)	Φ(τ)	τ	φ(τ)	Φ(τ)	τ	φ(τ)	Φ(τ)
·00	·3989423	·5000000						
·01	·3989223	·5039894	·51	·3502919	·6949743	1·01	·2395511	·8437524
·02	·3988625	·5079783	·52	·3484925	·6984682	1·02	·2371320	·8461358
·03	·3987628	·5119665	·53	·3466677	·7019440	1·03	·2347138	·8484950
·04	·3986233	·5159534	·54	·3448180	·7054015	1·04	·2322970	·8508300
·05	·3984439	·5199388	·55	·3429439	·7088403	1·05	·2298821	·8531409
·06	·3982248	·5239222	·56	·3410458	·7122603	1·06	·2274696	·8554277
·07	·3979661	·5279032	·57	·3391243	·7156612	1·07	·2250599	·8576903
·08	·3976677	·5318814	·58	·3371799	·7190427	1·08	·2226535	·8599289
·09	·3973298	·5358564	·59	·3352132	·7224047	1·09	·2202508	·8621434
·10	·3969525	·5398278	·60	·3332246	·7257469	1·10	·2178522	·8643339
·11	·3965360	·5437953	·61	·3312147	·7290691	1·11	·2154582	·8665005
·12	·3960802	·5477584	·62	·3291840	·7323711	1·12	·2130691	·8686431
·13	·3955854	·5517168	·63	·3271330	·7356527	1·13	·2106856	·8707619
·14	·3950517	·5556700	·64	·3250623	·7389137	1·14	·2083078	·8728568
·15	·3944793	·5596177	·65	·3229724	·7421539	1·15	·2059363	·8749281
·16	·3938684	·5635595	·66	·3208638	·7453731	1·16	·2035714	·8769756
·17	·3932190	·5674949	·67	·3187371	·7485711	1·17	·2012135	·8789995
·18	·3925315	·5714237	·68	·3165929	·7517478	1·18	·1988631	·8809999
·19	·3918060	·5753454	·69	·3144317	·7549029	1·19	·1965205	·8829768
·20	·3910427	·5792597	·70	·3122539	·7580363	1·20	·1941861	·8849303
·21	·3902419	·5831662	·71	·3100603	·7611479	1·21	·1918602	·8863606
·22	·3894038	·5870644	·72	·3078513	·7642375	1·22	·1895432	·8887676
·23	·3885286	·5909541	·73	·3056274	·7673049	1·23	·1872354	·8906514
·24	·3876166	·5948349	·74	·3033893	·7703500	1·24	·1849373	·8925123
·25	·3866681	·5987063	·75	·3011374	·7733726	1·25	·1826491	·8943502
·26	·3856834	·6025681	·76	·2988724	·7763727	1·26	·1803712	·8961653
·27	·3846627	·6064199	·77	·2965948	·7793501	1·27	·1781038	·8979577
·28	·3836063	·6102612	·78	·2943050	·7823046	1·28	·1758474	·8997274
·29	·3825146	·6140919	·79	·2920038	·7852361	1·29	·1736022	·9014747
·30	·3813878	·6179114	·80	·2896916	·7881446	1·30	·1713686	·9031995
·31	·3802264	·6217195	·81	·2873689	·7910299	1·31	·1691468	·9049021
·32	·3790305	·6255158	·82	·2850364	·7938919	1·32	·1669370	·9065825
·33	·3778007	·6293000	·83	·2826945	·7967306	1·33	·1647397	·9082409
·34	·3765372	·6330717	·84	·2803438	·7995458	1·34	·1625551	·9098773
·35	·3752403	·6368307	·85	·2779849	·8023375	1·35	·1603833	·9114920
·36	·3739106	·6405764	·86	·2756182	·8051055	1·36	·1582248	·9130850
·37	·3725483	·6443088	·87	·2732444	·8078498	1·37	·1560797	·9146565
·38	·3711539	·6480273	·88	·2708640	·8105703	1·38	·1539483	·9162067
·39	·3697277	·6517317	·89	·2684774	·8132671	1·39	·1518308	·9177356
·40	·3682701	·6554217	·90	·2660852	·8159399	1·40	·1497275	·9192433
·41	·3667817	·6590970	·91	·2636880	·8185887	1·41	·1476385	·9207302
·42	·3652627	·6627573	·92	·2612863	·8212136	1·42	·1455641	·9221962
·43	·3637136	·6664022	·93	·2588805	·8238145	1·43	·1435046	·9236415
·44	·3621349	·6700314	·94	·2564713	·8263912	1·44	·1414600	·9250663
·45	·3605270	·6736448	·95	·2540591	·8289439	1·45	·1394306	·9264707
·46	·3588903	·6772419	·96	·2516443	·8314724	1·46	·1374165	·9278550
·47	·3572253	·6808225	·97	·2492277	·8339768	1·47	·1354181	·9292191
·48	·3555325	·6843863	·98	·2468095	·8364569	1·48	·1334353	·9305634
·49	·3538124	·6879331	·99	·2443904	·8389129	1·49	·1314684	·9318879
·50	·3520653	·6914625	1·00	·2419707	·8413447	1·50	·1295176	·9331928

TABLE I (*Contd.*)

τ	φ(τ)	Φ(τ)	τ	φ(τ)	Φ(τ)	τ	φ(τ)	Φ(τ)
1·51	·1275830	·9344783	2·01	·0529192	·9777844	2·51	·0170947	·9939634
1·52	·1256646	·9357445	2·02	·0518636	·9783083	2·52	·0166701	·9941323
1·53	·1237628	·9369916	2·03	·0508239	·9788217	2·53	·0162545	·9942969
1·54	·1218775	·9382198	2·04	·0498001	·9793248	2·54	·0158476	·9944574
1·55	·1200090	·9394292	2·05	·0487920	·9798178	2·55	·0154493	·9946139
1·56	·1181573	·9406201	2·06	·0477996	·9803007	2·56	·0150596	·9947664
1·57	·1163225	·9417924	2·07	·0468226	·9807738	2·57	·0146782	·9949151
1·58	·1145048	·9429466	2·08	·0458611	·9812372	2·58	·0143051	·9950600
1·59	·1127042	·9440826	2·09	·0449148	·9816911	2·59	·0139401	·9952012
1·60	·1109208	·9452007	2·10	·0439836	·9821356	2·60	·0135830	·9953388
1·61	·1091548	·9463011	2·11	·0430674	·9825708	2·61	·0132337	·9954729
1·62	·1074061	·9473839	2·12	·0421661	·9829970	2·62	·0128921	·9956035
1·63	·1056748	·9484493	2·13	·0412795	·9834142	2·63	·0125581	·9957308
1·64	·1039611	·9494974	2·14	·0404076	·9838226	2·64	·0122315	·9958547
1·65	·1022649	·9505285	2·15	·0395500	·9842224	2·65	·0119122	·9959754
1·66	·1005864	·9515428	2·16	·0387069	·9846137	2·66	·0116001	·9960930
1·67	·0989255	·9525403	2·17	·0378779	·9849966	2·67	·0112951	·9962074
1·68	·0972823	·9535213	2·18	·0370629	·9853713	2·68	·0109969	·9963189
1·69	·0956568	·9544860	2·19	·0362619	·9857379	2·69	·0107056	·9964274
1·70	·0940491	·9554345	2·20	·0354746	·9860966	2·70	·0104209	·9965330
1·71	·0924591	·9563671	2·21	·0347009	·9864474	2·71	·0101428	·9966358
1·72	·0908870	·9572838	2·22	·0339408	·9867906	2·72	·0098712	·9967359
1·73	·0893326	·9581849	2·23	·0331939	·9871263	2·73	·0096058	·9968333
1·74	·0877961	·9590705	2·24	·0324603	·9874545	2·74	·0093466	·9969280
1·75	·0862773	·9599408	2·25	·0317397	·9877755	2·75	·0090936	·9970202
1·76	·0847764	·9607961	2·26	·0310319	·9880894	2·76	·0088465	·9971099
1·77	·0832932	·9616364	2·27	·0303370	·9883962	2·77	·0086052	·9971972
1·78	·0818278	·9624620	2·28	·0296546	·9886962	2·78	·0083697	·9972821
1·79	·0803801	·9632730	2·29	·0289847	·9889893	2·79	·0081398	·9973646
1·80	·0789502	·9640697	2·30	·0283270	·9892759	2·80	·0079155	·9974449
1·81	·0775379	·9648521	2·31	·0276816	·9895559	2·81	·0076965	·9975229
1·82	·0761433	·9656205	2·32	·0270481	·9898296	2·82	·0074829	·9975988
1·83	·0747663	·9663750	2·33	·0264265	·9900969	2·83	·0072744	·9976726
1·84	·0734068	·9671159	2·34	·0258166	·9903581	2·84	·0070711	·9977443
1·85	·0720649	·9678432	2·35	·0252182	·9906133	2·85	·0068728	·9978140
1·86	·0707404	·9685572	2·36	·0246313	·9908625	2·86	·0066793	·9978818
1·87	·0694333	·9692581	2·37	·0240556	·9911060	2·87	·0064907	·9979476
1·88	·0681436	·9699460	2·38	·0234910	·9913437	2·88	·0063067	·9980116
1·89	·0668711	·9706210	2·39	·0229374	·9915758	2·89	·0061274	·9980738
1·90	·0656158	·9712834	2·40	·0223945	·9918025	2·90	·0059525	·9981342
1·91	·0643777	·9719334	2·41	·0218624	·9920237	2·91	·0057821	·9981929
1·92	·0631566	·9725711	2·42	·0213407	·9922397	2·92	·0056160	·9982498
1·93	·0619524	·9731966	2·43	·0208294	·9924506	2·93	·0054541	·9983052
1·94	·0607652	·9738102	2·44	·0203284	·9926564	2·94	·0052963	·9983589
1·95	·0595947	·9744119	2·45	·0198374	·9928572	2·95	·0051426	·9984111
1·96	·0584409	·9750021	2·46	·0193563	·9930531	2·96	·0049929	·9984618
1·97	·0573038	·9755808	2·47	·0188850	·9932443	2·97	·0048470	·9985110
1·98	·0561831	·9761482	2·48	·0184233	·9934309	2·98	·0047050	·9985588
1·99	·0550789	·9767045	2·49	·0179711	·9936128	2·99	·0045666	·9986051
2·00	·0539910	·9772499	2·50	·0175283	·9937903	3·00	·0044318	·9986501